Constraint Programming 433-637

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Constraint programming

Finding the “best” solution from a HUGE set of alternatives

Examples

- solving Sudoku problems
- planning a mining operation
- designing an airplane

Sudoku

- How many ways can you fill a Sudoku board with numbers 1-9?

- How many Sudoku puzzles are there?

5	9	3	7	6	2	8	1	4
2	6	8	4	3	1	5	7	9
7	1	4	9	8	5	2	3	6
3	2	6	8	5	9	1	4	7
1	8	7	3	2	4	9	6	5
4	5	9	1	7	6	3	2	8
9	4	2	6	1	8	7	5	3
8	3	5	2	4	7	6	9	1
6	7	1	5	9	3	4	8	2

6,670,903,752,021,072,936,960

Learning Objectives

- Understand
  - what “combinatorial” problems are
  - the main technologies used for solving them
- Learn
  - the distinction between modelling and solving a problem
  - strengths and weaknesses of the solving technologies
  - different ways of achieving scalability in a solution
  - which problems should use which solving technologies

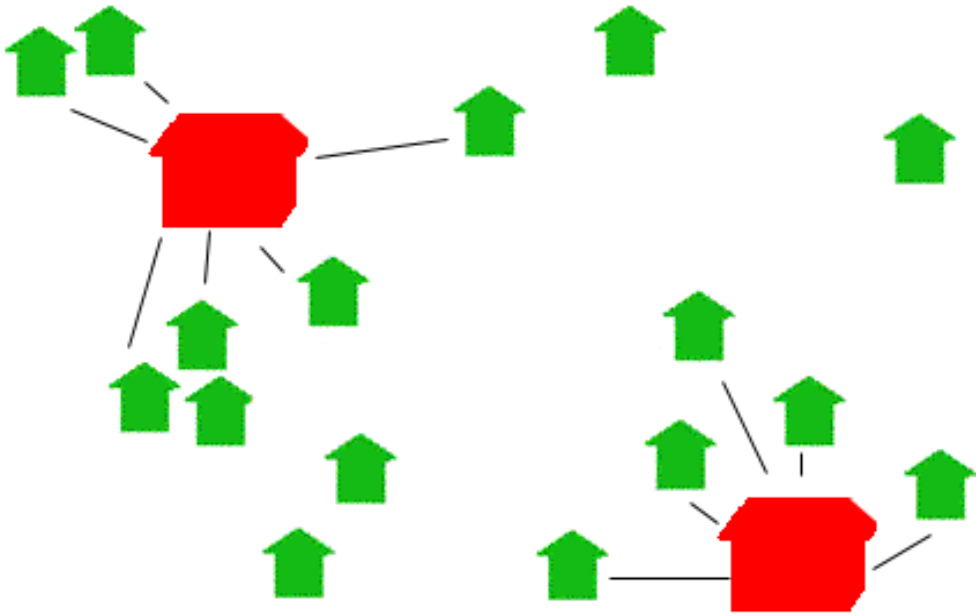
Textbooks

- Prescribed texts:
  - None
- Recommended texts:
  - MiniZinc (modelling language) tutorial and documentation
    - An informal guide free and accessible from LMS
  - Programming with Constraints: an Introduction. Kim Marriott and Peter J. Stuckey, MIT Press. 1998.
  - Operations Research: Applications and Algorithms. Wayne L. Winston, Brooks Cole, 1998.
  - Principles of Constraint Programming. Krzysztof Apt. Cambridge. 2003.

Combinatorial Problems in the Real World

- Combinatorial Problems
  - Where we have to choose amongst a set of decisions
  - Are ubiquitous
  - Good answers can save a great deal of money, pollution, carbon, etc
- They appear at all levels of an organization
  - Strategic (Typically years)
    - Where to build factories/distribution centres
    - How many aircraft to order
  - Tactical (Typically weeks or months)
    - What price to set
    - How many trips per day to schedule
  - Operational (Typically today or tomorrow)
    - How to handle priority jobs
    - How to recover when a pilot/doctor is absent

Strategic Planning: new distribution centres to serve outlets

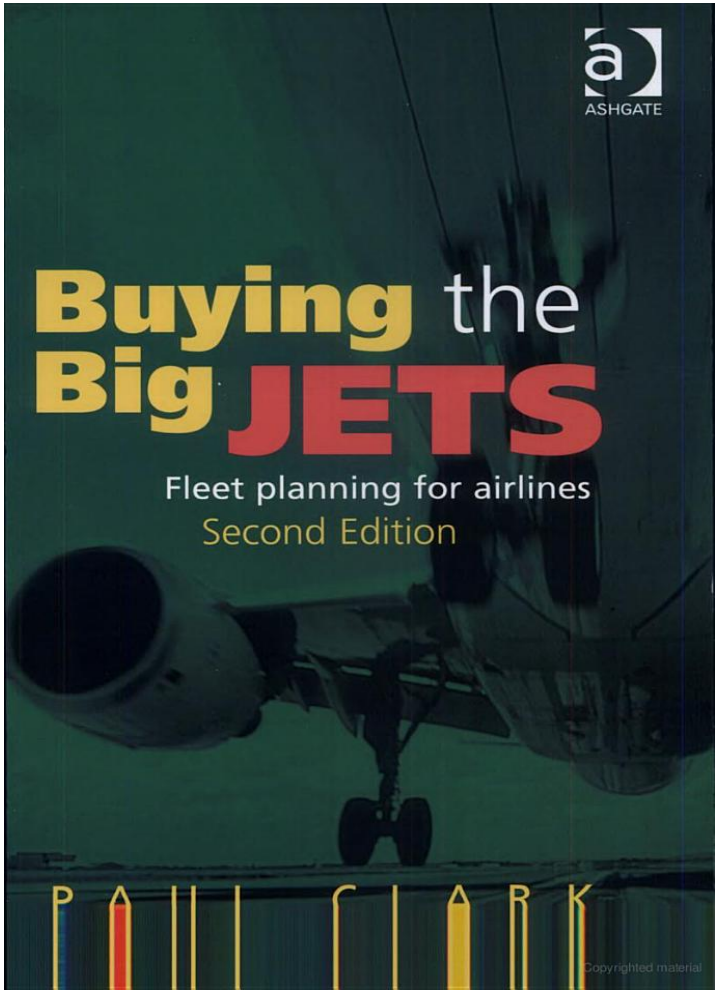


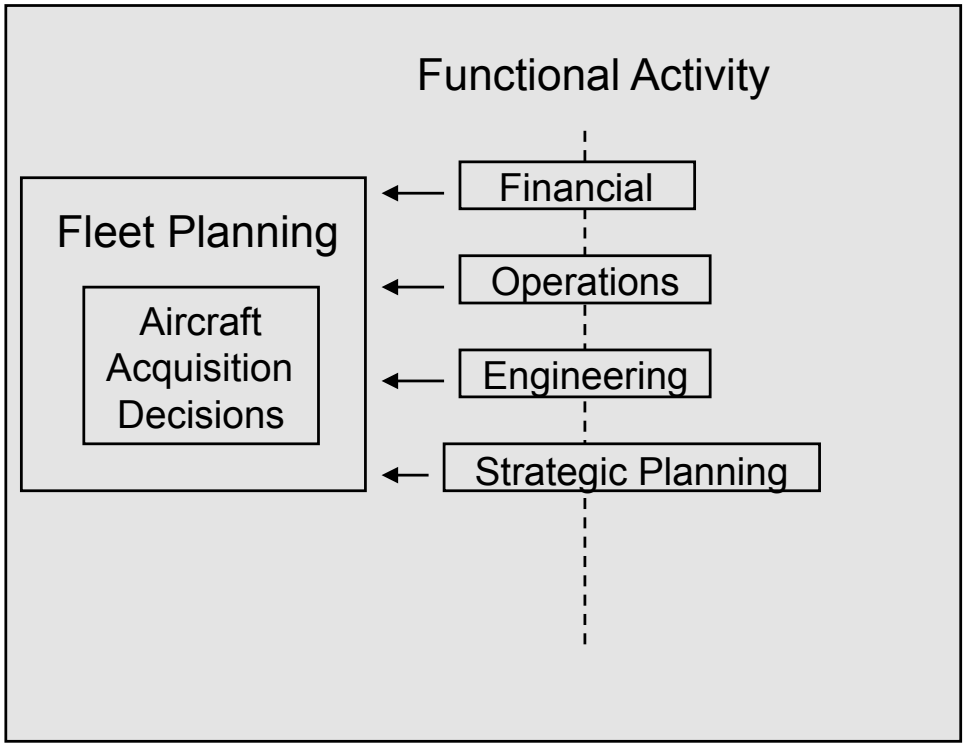
Which outlets will there be in a years time?

What land will be available with what building permissions?

How long will they take to construct and how is this impacted by the DC capacity?

Strategic Planning: buying new aircraft to meet demand





Strategic planning: road planning and design

Main road planning goals

- Safer communities
- Industry competitiveness
- Liveable communities
- Environmental conservation

Strategic Planning - characteristics

- Decisions - involve major commitments
  - Money
  - Time
  - Manpower and Resources
- Timescales - long
  - Years rather than weeks
- Outcomes - fundamental to the future of the organisation
  - The right decision will make a LOT of money
  - The right decision will delight the customers
  - The right decision will create further growth opportunities
- Choices - broad and loose
  - The number and nature of the options may be unclear
  - The required commitments may be imprecise
  - The payoffs may be a little vague

Tactical planning – flight leg pricing

- The airline industry is probably the industry that for which many of the revenue management concepts have been introduced.
- Each flight (leg) has 26 ticket classes
- Tickets for a single class may be sold at different prices
- Revenue management systems include a forecaster (estimating the future potential sales of tickets from now till the day of departure) and an optimizer (defining the appropriate inventory controls from the forecasted demand).

Tactical planning: lecture theatres and times

Allocate+

Subject	Description	Faculty	Group	#Activities	Places	Enrolments
FIT3015_CA_S1_DAY	IE PROJECT	50000566	Lecture	1	10	27
FIT3036_CL_S2_DAY	COMP SCI PROJECT	50000566	Support-Class	1	18	
FIT3047_SA_S1_DAY	IE PROJECT	50000566	Comp-Lab	1	24	37
FIT3066_CA_S1_DAY	IT STRATEGY & MGT	50000566	Tutorial	2	41	60
FIT3031_CA_S1_DAY	INFO & NETWORK SEC	50000566	Laboratory	4	69	100
FIT2034_CA_S1_DAY	PROGRAMMING 2	50000566	Lecture	1	60	85
FIT2048_CA_S1_DAY	GAME TECH	50000566	Laboratory	3	48	67
FIT3008_BE_S1_DAY	DIG VIDEO POST PROD	50000566	Lecture	1	20	27

- Allocate a lecture theatre of the right size to each lecture
- Don’t put two lectures in the same theatre at the same time
- Timetable two lectures in the same unit to follow each other if required
- Ensure theatres have the facilities required for each unit allocated

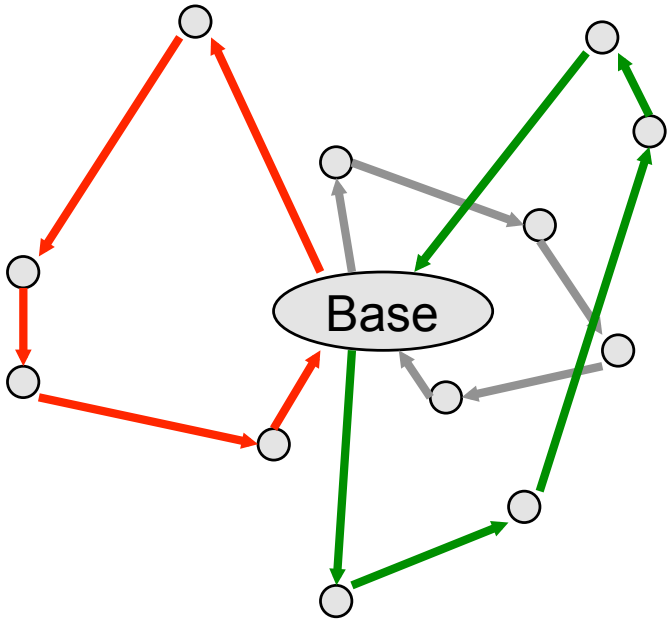
Tactical planning - characteristics

- Decisions - involve known resources
  - aircraft
  - lecture theatres
- Timescales - medium
  - Months or weeks
- Outcomes - significant
  - The right schedule is economical
  - The right schedule won’t disappoint the customers
- Choices - optimise chosen goals under clear constraints
  - Revenue
  - Cost
  - Resource usage

Operational Planning - onsite engineers

Allocating sites to engineers (cont.)

Clients:



Operational planning - characteristics

- Decisions - involve a current set of tasks and known resources
  - allocate tasks to resources
  - re-allocate and re-schedule after events
- Timescales - short
  - Today or tomorrow (or instantaneously!)
- Outcomes – short-term
  - impact on today's customers
- Choices - minimise disruption
  - Try to complete tasks
  - Minimise delay overtime or other extra costs
  - Get back on schedule (for tomorrow…)

Capturing the problem

- Understanding what the real problem is that has to be solved can be
  - Challenging
  - Require “deep communication” with the people who solve it now
  - Wont be right the first time

The Holy Grail for Constraint Programming

- Model Problems Naturally
  - constraints
  - solution properties
- Solve them efficiently
  - overcome combinatorial explosion
- Compile
  - Natural models to efficient solutions

Summary

- Combinatorial optimizations problems
  - Are everywhere
  - Are important
  - And are difficult to solve
- Capturing a real world problem
  - Is difficult
  - Filled with lots of choices
  - In reality needs a feedback loop with the end user