

## *Feuille de travaux dirigés n° 1*

# Constraint Programming

### **Exercice 1.1**

If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23. Propose a CSP to find the sum of all the multiples of 3 or 5 below 1000.

### **Exercice 1.2**

Consider a 11 by 11 binary matrix. The goal is to put 11 ones in each row in such a way that each column has 11 ones, and each pair of rows must have exactly 1 one in the same column. Propose a CSP to solve this problem.

### **Exercice 1.3**

These problems are said to have many practical applications including sensor placements for x-ray crystallography and radio astronomy. A Golomb ruler may be defined as a set of  $m$  integers  $0 = a_1 < a_2 < \dots < a_m$  such that the  $m(m-1)/2$  differences  $a_j - a_i, 1 \leq i < j \leq m$  are distinct. Such a ruler is said to contain  $m$  marks and is of length  $a_m$ . The objective is to find optimal (minimum length) or near optimal rulers. Note that a symmetry can be removed by adding the constraint that  $a_2 - a_1 < a_m - a_{m-1}$ , the first difference is less than the last.

There is no requirement that a Golomb ruler measures all distances up to its length - the only requirement is that each distance is only measured in one way. However, if a ruler does measure all distances, it is classified as a perfect Golomb ruler. For example, 0, 1, 3, 6 is not a Golomb ruler while 0, 1, 4, 6 is one.

There exist several interesting generalizations of the problem which have received attention like modular Golomb rulers (differences are all distinct mod a given base), disjoint Golomb rulers, Golomb rectangles (the 2-dimensional generalization of Golomb rulers), and difference triangle sets (sets of rulers with no common difference).

For a related problem, please see Costas Arrays.