Tutorial #1

Functional Dependencies

G. Raschia

Last update: January 21, 2024

Problem 1

Consider a relation with schema $R(A, B, C, D)$ and FD's $AB \to C$, $C \to D$, and $D \to A$.

1. What are all the nontrivial FD's that follow from the given FD's? You should restrict yourself to FD's with single attributes on the right side.

   Solution: There are 11 FD's to find. $AB \to D$ and $C \to A$, by transitivity, but also $ABC \to D$, $ABD \to C$, $AC \to D$, $BC \to D$, $ABC \to D$, $BD \to A$, $CD \to A$, $BCD \to A$ by augmentation. And ultimately, $BD \to C$, by pseudo-transitivity.

2. What are all the keys of R?

   Solution: B belongs to any key but it is not a superkey itself. $AB^+ = ABCD$, $BC^+ = BCDA$, $BD^+ = BDAC$. Then keys are AB, BC, BD.

3. What are all the superkeys for R that are not keys?

   Solution: They are all the supersets of any key, thus ABC, ABD, BCD, $ABCD$.

Problem 2

1. Show that each of the following are not valid rules about FD's by giving example relations that satisfy the given FD's (following the ”if”) but not the FD that allegedly follows (after the “then”):
   (a) If $A \to B$ then $B \to A$
   (b) If $AB \to C$ and $A \to C$, then $B \to C$

Solution: With the Inference Test, one easily obtain:

(a) $\{(0,0),(1,0)\}$

(b) $\{(0,0,0),(1,0,1)\}$

2. By using the 3+3 Armstrong's Axioms only, show that

$$\{AB \to CD, C \to EH, D \to G\} \models AB \to EHG$$

Solution: 1. $AB \to CD$ gives $AB \to C$ and $AB \to D$ by decomposition
2. $AB \to C$ (1) and $C \to EH$ gives $AB \to EH$ by transitivity
3. $AB \to D$ (1) and $D \to G$ gives $AB \to G$ by transitivity
4. (2) and (3) gives $AB \to EHG$ by union.

Problem 3

Suppose we have relation $R(A,B,C,D,E)$, with some set of FD's, and we wish to project those FD's onto relation $S(A,B,C)$.

1. Give the FD's that hold in S if the FD's for R are
   (a) $AB \to DE$, $C \to E$, $D \to C$, and $E \to A$
   (b) $A \to D$, $BD \to E$, $AC \to E$, and $DE \to B$

Solution:

(a) Compute $A^+ = A$, $B^+ = B$, $C^+ = CEA$, $AB^+ = ABCDE$, $AC^+ = ACE$, $BC^+ = BCEAD$. Then FD's in S are $C \to A$ and $AB \to C$ ($BC \to A$ holds but it comes after $C \to A$).

(b) Compute $A^+ = AD$, $B^+ = B$, $C^+ = C$, $AB^+ = ABDE$, $AC^+ = ACEDB$, $BC^+ = BC$. Then the only FD in S is $AC \to B$.