Tutorial #2
Normal Forms

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Problem 1

Given a relation $R(A, B, C, D)$ with the FD's $AB \to C$, $C \to D$, and $D \to A$.

1. Show all the BCNF violations. Do not forget to consider FD's that are not in the given set, but follow from them. However, it is not necessary to give violations that have more than one attribute on the right side.

> **Solution:** Keys are AB, BC and BD. Then, $C \to D$ and $D \to A$ both violate BCNF. But also, one must check for $A^+ = A$, $B^+ = B$, $C^+ = CDA$ (then C→ A) and $D^+ = DA$.
>
> Also, one checks $AC^+ = ACD$ (then $AC \to D$) $AD^+ = ADC$ (then $AD \to C$) and $CD^+ = CDA$ (then $CD \to A$).

2. Is R 3NF?

> **Solution:** Yes. All the attributes are prime, then none of the existing FD's violates 3NF definition.

3. Decompose the relation R, as necessary, into collections of relations that are in BCNF.

> **Solution:** Taking $C \to D$ as the pivot FD: $R_1(CDA)$ and $R_2(CB)$. $\Sigma_1 = \{C \to D, D \to A\}$ and $\Sigma_2 = \emptyset$. R_2 is obviously BCNF. C is key in R_1, hence $D \to A$ violates BCNF.
>
> One decomposes into $R_{11}(\underline{D}A)$ and $R_{12}(\underline{C}D)$ both BCNF.

Problem 2

Let $R(A, B, C, D, E)$ be decomposed into relations with the following three sets of attributes: $\{A, B, C\}$, $\{B, C, D\}$, and $\{A, C, E\}$.

1. For each of the following sets of FD's, use the Chase Test to tell whether the decomposition of R is lossless. For those that are not lossless, give an example of an instance of R that returns more than R when projected onto the decomposed relations and rejoined.
   (a) $AC \to E$ and $BC \to D$
   (b) $A \to D$, $CD \to E$, and $E \to D$

Solution:

(a)

number	A	B	C	D	E
t_1	a	b	c	d_1	e_1
t_2	a_2	b	c	d	e_2
t_3	a	b_3	c	d_3	e

By $AC \to E$ one get $t_1.E = e$ and by $BC \to D$, $t_1.D = d$. Then $t_1 = t$ and we proved the losseless join decomposition.

(b) The initial table is the same as before. Firing $A \to D$ yields to $t_3.D = d_1$. Then, $CD \to E$ brings $t_1.E = e$. And $E \to D$ cannot help.

Finally, we stay with the following table:

number	A	B	C	D	E
t_1	a	b	c	d_1	e
t_2	a_2	b	c	d	e_2
t_3	a	b_3	c	d_1	e

that shows a counter-example of a R table where the tuple $t = (abcde)$ comes from the rejoin whereas it is not in the table.

2. For each of the previous sets of FD's, are dependencies preserved by the decomposition?

Solution:

(a) yes, trivially.

(b) No, trivially.

1 Problem 3

1. Compute the minimal cover of the following set of FD's:

$$\{AB \to CF, BG \to C, AEF \to C, ABG \to ED, CF \to AE, A \to CG, AD \to FEG, AC \to B\}$$

Solution: First, one split rhs into singleton attributes (obvious). Then, one must discard redundant FD's. To this end, it is worth to note that $A^+ = ABCDEFG$! Then, we have a shortcut to $\mathcal{F}_2 = \{A \to C, A \to F, BG \to C, A \to E, A \to D, CF \to A, CF \to E, A \to G, A \to B\}$.

$BG \to C$ cannot be removed since $C \notin BG+_{\mathcal{F}_2 - \{BG \to C\}} = BG$. $CF \to A$ neither. But $CF \to E$ can be thanks to $CF \to A$ and $A \to E$.

In the third step, one has to reduce composite lhs of FD's. For instance, is B extraneous in $BG \to C$? The answer is no since $G^+ = G$ ($C \notin G^+$). idem for G wrt. B^+. And $CF \to A$ is also irreducible.

Then, the definitive minimal cover is

$$\mathcal{F}_{\min} = \{A \to BCDEFG, BG \to C, CF \to A\}$$

As a side note, one can see that the schema is 3NF since the keys are A, CF and BGF and DF's are either from a (super)key ($A \to BCDEFG$ and $CF \to A$), or to a prime attribute ($BG \to C$).

Problem 4

Consider the relation `Courses`(C, T, H, R, S, G), whose attributes may be thought of informally as course, teacher, hour, room, student, and grade.

1. Translate into FD's, when applicable, the following constraints:
   (a) a course has a unique teacher;
   (b) only one course can meet in a given room at a given hour;
   (c) a teacher can be in only one room at a given hour;
   (d) a student can be in only one room at a given hour;
   (e) students get only one grade in a course.

Solution:

(a) $C \to T$

(b) $RH \to C$

(c) $TH \to R$

(d) $SH \to R$

(e) $SC \to G$

2. What are all the keys for `Courses`?

> **Solution:** H and S belong to any key. $HS^+ = HSRCTG$. That is the key.

3. Verify that the given FD's are their own minimal basis.

> **Solution:** No FD is redundant. And by construction (semantics), no lhs attribute can be removed.

4. Use the 3NF synthesis algorithm to find a lossless-join, dependency-preserving decomposition of `Courses` into 3NF relations. Is there any of the relations not in BCNF?

> **Solution:** Following the minimum basis, one get:
>
> - $\underline{C}T$
> - $\underline{RH}C$
> - $\underline{TH}R$
> - $\underline{SH}R$ (includes the HS key)
> - $\underline{SC}G$
>
> All those relations have a unique FD from the key. They are BCNF.