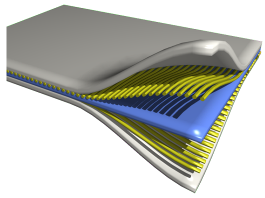
UNIVERSITÉ DE NANTES



Composite Materials examination : correction

PC and CTMF options.

December 5^{th} 2018 - Nb of pages **1** - *Authorized documents: 1 double-sided A4 sheet of notes*

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1 Knowledge questions (7pts)

1. In the UD case load along fibers, a parallel model is natural for stresses, so that :

$$\sigma_c = V_F \sigma_f + (1 - V_F)\sigma_m \tag{1}$$

The maximum resistance of a material is a stress value, so it follows the equation (1). The problems arises due to the low elongation at break of fibers $(A\%)_f$. Fracture occurs when $\sigma_f = E_f \times (A\%)_f = (R_m)_f$. At this moment, the stress in the matrix (assuming a higher deformability) reaches $\sigma_m(A\%)_f)$, which is not necessarily in the elastic range. One can thus write :

$$(R_m)_c = V_F (R_m)_f + (1 - V_F)\sigma_m(A\%)_f \tag{2}$$

An improvement in the matrix resistance is obtained only if this value is higher than $(R_m)_m$, so that we can write the following inequation :

$$V_F (R_m)_f + (1 - V_F)\sigma_m(A\%)_f > (R_m)_m \tag{3}$$

For an improvement to be obtained on the maximum resistance of the material, a minimal volume fraction of fiber V_F^* can therefore be defined :

$$V_F > V_F^* = \frac{(R_m)_m - \sigma_m(A\%)_f}{(R_m)_f - \sigma_m(A\%)_f} \tag{4}$$

2. The plane stress assumption in the Kirchhoff-Love plates theory is justified by two argument : (i) the small thickness of plates compared to the in-plane dimensions and (ii) the absence of mechanical loading in the the perpendicular direction to the plane of the plate.
3. Integrated stresses :

$$\{\hat{N}\} = \int_{z=-h/2}^{z=h/2} \left\{ \begin{array}{c} \sigma_{xx} \\ \sigma_{yy} \\ \sqrt{2}\sigma_{xy} \end{array} \right\} dz$$

Integrated moments :

$$\{\hat{M}\} = \int_{z=-h/2}^{z=h/2} z \times \left\{ \begin{array}{c} \sigma_{xx} \\ \sigma_{yy} \\ \sqrt{2}\sigma_{xy} \end{array} \right\} dz$$

4. The aspect ratio, defined as the length l to diameter d ratio of fibers :

$$a_r = \frac{l}{d}$$

5. 5 material constants.
6. In the case of a in-plane loading, when tensile load is applied in any direction with respect to the orthotropy (natural) axes, a shear perpendicular to the tensile direction is observed (shear/tensile coupling).

2. Optimal design of a plate submitted to a tensile loading (13pts)

We consider a planar layered composite beam made of $2n$ layers of thickness e. The lay-up is chosen with alternate orientations $+45°$ and $-45°$. The cross section of the composite beam is of width b and height $h = 2Ne$, its length being denoted L. The beam is clamped at one edge and submitted to a tensile force F in the $0°$ direction at the other edge. The design objective is to obtain a displacement lower than a certain value u_{max} at the loaded edge.

<u>Data :</u>

- properties of one ply : $E_L = 110$ GPa ; $E_T = 20$ GPa ; $\nu_{LT} = 0,3$; $G_{LT} = 12,5$ GPa.
- L=1 m, b=25 mm, e=0,16mm, F=5kN, u_{max}=1mm.

1. Calculate the coefficients of the planar stiffness matrices $[Q^{45}]$ and $[Q^{-45}]$. In the natural axes (0°) :

$$[Q^0] = \begin{bmatrix} 111.8 & 6.1 & 0 \\ 6.1 & 20.3 & 0 \\ 0 & 0 & 25.0 \end{bmatrix} \text{GPa}$$

   Applying the rotation formulas gives for both orientation :

$$[Q^{45}] = \begin{bmatrix} 48.6 & 23.6 & 32.3 \\ 23.6 & 48.6 & 32.3 \\ 32.3 & 32.3 & 60.0 \end{bmatrix} \text{GPa} \quad \text{and} \quad [Q^{-45}] = \begin{bmatrix} 48.6 & 23.6 & -32.3 \\ 23.6 & 48.6 & -32.3 \\ 32.3 & 32.3 & 60.0 \end{bmatrix} \text{GPa}$$

2. $[A]_0 = e\left([Q^{45}] + [Q^{-45}]\right)$, so :

$$[A]_0 = \begin{bmatrix} 15.5 & 7.5 & 0 \\ 7.5 & 15.5 & 0 \\ 0 & 0 & 19.2 \end{bmatrix} \text{kN.mm}^{-1}$$

3.

$$\{\hat{N}\} = \int_{z=-h/2}^{z=h/2} \left\{ \begin{array}{c} \sigma_{xx} \\ \sigma_{yy} \\ \sqrt{2}\sigma_{xy} \end{array} \right\} dz = \left\{ \begin{array}{c} F/b \\ 0 \\ 0 \end{array} \right\} = \left\{ \begin{array}{c} 2 \\ 0 \\ 0 \end{array} \right\} \text{kN.mm}^{-1}$$

4. In the uncoupled situation, one can easily calculate $\{\varepsilon_m\}_0$ by :

$$\{\varepsilon_m\}_0 = [A]_0^{-1}\{\hat{N}\}$$

   which leads to

$$\{\varepsilon_m\}_0 = \left\{ \begin{array}{c} 16,8\% \\ -8,2\% \\ 0 \end{array} \right\}$$

5. According to the design constraint $u_max < 1$mm, the maximal admissible strain is

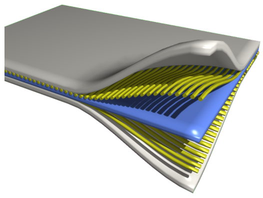
$$\varepsilon_{xx,max} = \frac{u_{max}}{L} = \frac{1}{1000} = 0,1\%$$

   As obvious from previous calculations, in the case of $2n$ layers, the integrated stiffness matrx $[A]$ will be

$$[A] = n \times [A]_0$$

   so that for the same loading as before, the resulting strain well be divided by a factor n. One thus has to satisfy :

$$\varepsilon_{xx} = \frac{\varepsilon_{xx,0}}{n} < \varepsilon_{xx,max}$$



Consequently

$$n > \frac{\varepsilon_{xx,0}}{\varepsilon_{xx,max}} = 16.83$$

The minimum number of layers is therefore **17**.

6. Using both the Kirchhof-Love kinematic assumptions and the orthotropic constitutive equations, one can write for eahc layer k

$$\{\sigma^k\} = [Q^k](\varepsilon_m + z\gamma)$$

In the absence of tension/bending coupling, as assumed here, γ can be proved to be null in this loading case. The stress is therefore constant in each layer so that we can compute it for one layer at 45 and one other at -45. One just has to calculate the membrane strain with $n = 17$ layers, which is just

$$\{\varepsilon_m\} = \{\varepsilon_m\}/17 = \left\{ \begin{array}{c} 1\% \\ -0.5\% \\ 0 \end{array} \right\}$$

Doing so, on obtains for all 45 and -45 layers :

$$\sigma^{45} = \left\{ \begin{array}{c} 368 \\ 0 \\ 165 \end{array} \right\} \text{MPa and } \sigma^{-45} = \left\{ \begin{array}{c} 368 \\ 0 \\ -165 \end{array} \right\}$$

7. Further discussion : the coupling matrix $[B]$ is never null for this kind of layup so that the assumption of uncoupling is not strictly speaking valid. One nevertheless feels that increasing the number of layers will rapidly decrease the importance of this effect.
   — Justify the first assertion : the layup is clearly non-symmetric
   — For each layer k, the layer located at $k+n$ has got the other orientation. Thus summing from 1 to n only, one can write from the general expression of $[B]$:

$$[B] = \sum_{k=1}^{n} e\frac{z_k + z_{k-1}}{2}\left([Q^{-45}] - [Q^{+45}]\right)$$

$$[B] = \left(\sum_{k=1}^{n} \frac{z_k + z_{k-1}}{2}\right) \begin{bmatrix} 0 & 0 & 10400 \\ 0 & 0 & 10400 \\ 10400 & 10400 & 0 \end{bmatrix}$$

   Observing that $\frac{z_k + z_{k-1}}{2} = e(\frac{1}{2} + (k-1)) = e(k - \frac{1}{2})$, one has

$$\sum_{k=1}^{n} \frac{z_k + z_{k-1}}{2} = e\sum_{k=1}^{n}(\frac{z_k + z_{k-1}}{2})k - \frac{1}{2}) = e(\frac{n(n+1)}{2} - \frac{n}{2}) = e\frac{n^2}{2}$$

$$[B] = \frac{n^2}{2} \begin{bmatrix} 0 & 0 & 1656 \\ 0 & 0 & 1656 \\ 16564 & 1656 & 0 \end{bmatrix}$$

   It does not decrease towards 0. The coefficients even rapidly increase (as n^2) with the number of layers.
   — If this layup is submitted to a tensile load as in the present example, there will be some torsion induced. Each pair of antisymmetric layer create a moment along the $\vec{x}$ axis, which is growing with the distance of the layer to the mid-plane.