Constraint Programming:
Formal Presentation

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Objectives

- formal definition of constraint and CSP
- equivalence of CSP's

Formal Presentation

Constraint : definition

Consider

- some variables $x_1, \ldots, x_k$
- some domains $D_1, \ldots, D_k$ associated to $x_1, \ldots, x_k$ respectively

a *constraint* C on $x_1, \ldots, x_k$ is a subset of $D_1 \times \ldots \times D_k$

domain D_i represents the possible values the variable x_i can take

Constraint : example

Consider variables x, y, z of domain $[0, 1]$

Consider the *constraint* and on x, y, z represents the Boolean and relation.

Then, and on x, y, z is a subset of $\{0, 1\} \times \{0, 1\} \times \{0, 1\}$ defined by :

$$\{(0, 0, 0), (0, 1, 0), (1, 0, 0), (1, 1, 1)\}$$

Constraint Satisfaction Problem : definition

Consider

- some variables $x_1, \ldots, x_n$
- some domains $D_1, \ldots, D_n$ associated to $x_1, \ldots, x_n$

intuitively : a *CSP* $\mathcal{P}$ is given by a set of constraints together with variables appearing in these constraints and their domains

formally : CSP

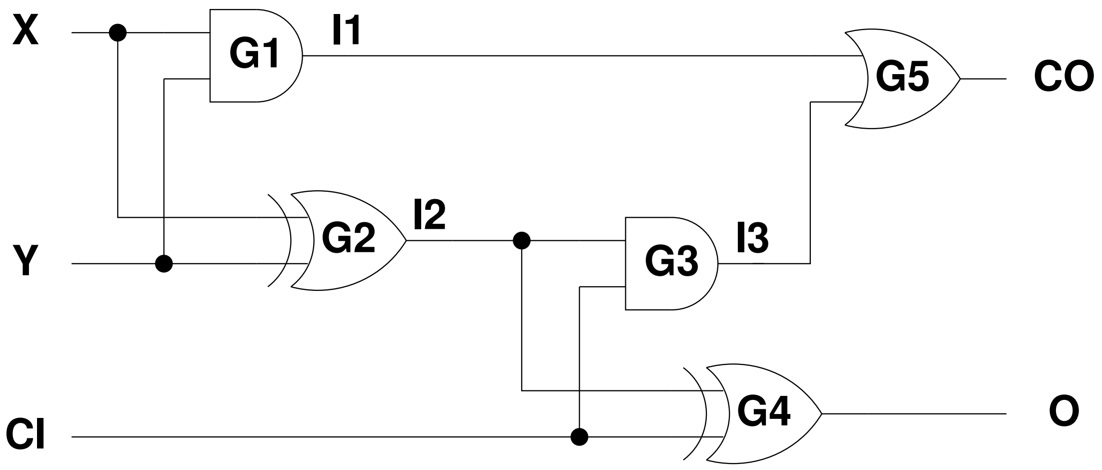
$$\mathcal{P} = \{C_1, \ldots, C_m;\ x_1 \in D_1, \ldots, x_n \in D_n\}$$

with each C_i on a subset of $x_1, \ldots, x_n$

$(d_1, \ldots, d_n)$ is a *solution* to $\mathcal{P}$ if for each constraint C_j on $x_{i_1}, \ldots, x_{i_{m_l}}$ $(i \in [1..l])$

$$(d_{i_1}, \ldots, d_{i_{m_l}}) \in C_j$$

CSP example : full-adder



$$\mathcal{P} = \{and(X,Y,I1), xor(X,Y,I2), and(I2,CI,I3),$$
$$xor(I2,CI,O), or(I1,I3,CO);$$
$$X \in [0,1], Y \in [0,1], CI \in [0,1], I1 \in [0,1],$$
$$I2 \in [0,1], I3 \in [0,1], CO \in [0,1], O \in [0,1]\}$$

solutions :

$$\{(0,0,0,0,0,0,0,0), (0,0,1,0,0,0,0,1), (0,1,0,0,1,0,0,1), (0,1,1,0,1,1,1,0),$$
$$(1,0,0,0,1,0,0,1), (1,0,1,0,1,1,1,0), (1,1,0,1,0,0,1,0), (1,1,1,1,0,0,1,1)\}$$